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Variable Sample Sizes for p-Charts

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Frequently when using fraction nonconforming control charts (i.e., p-charts) the practitioner finds that the sample size varies. For example, suppose we were to take a sample of 50 units for inspection from each lot and plot the fraction nonconforming found in the sample. Now our sampling program seems to be running smoothly when two units in one sample are lost or destroyed and can't be replaced. So this sample has only 48 units. Of course we can still inspect the sample and compute p, but the control limits for this sample are affected by the change in sample size (n), because the control limits formulas are:

$$\mathrm{UCLp} = \overline{\mathrm{p}} + 3\sqrt{\overline{\mathrm{p}}(1-\overline{\mathrm{p}})/\mathrm{n}}$$

$$\mathrm{LCLp} = \max\{0, \overline{\mathrm{p}} - 3\sqrt{\overline{\mathrm{p}}(1-\overline{\mathrm{p}})/\mathrm{n}}\}$$

So how do we solve this problem? There are four possible solutions:

1. We can compute the p value and the new control limits for each sample and plot them. However, when this approach is adopted the control chart limits will move up and down. Thus making it hard to apply the pattern test runs rules (e.g., two out of three points in zone A).

2. Ignore the problem if the difference between the actual and expected sample size is small (say less than 5%). This situation can probably be tolerated as long as the frequency of occurrence is low.

3. Compute the average sample size ($\overline{\mathrm{n}}$) and use this as the value in computing the control limits. Several texts suggest that this approach is acceptable as long as the difference in sample sizes does not exceed ± 20%. This recommendation is designed to deal with the case where the frequency of occurrence is high and/or the difference between sample sizes is significant.

4. The final choice is to compute standardized p values (z_p) for each samples p_i and n_i given by the formula:

$$z(p_i) = (p_i - \overline{p}) / \sqrt{\overline{p}(1-\overline{p})/n_i}$$

Then plot these z_p values on a standardized p-chart. This solution has both good and bad points. The good point is that it is more accurate than methods 1, 2 and 3. Hence it allows for accurate implementation of the Western Electric runs tests which are compromised by solution 1, 2 and 3. However, though the inferences regarding stability may be more accurate the intuitiveness of the chart is compromised. The values on the p-chart now map into standardized units which makes interpretation more difficult (i.e., $CLz_p = 0$, $UCLz_p = 3$, $LCLz_p = -3$). Thus, by looking at the z_p control chart we cannot easily tell what the fraction nonconforming was in a particular sample unless we know the sample size and the average nonconformance level of the process.

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