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The Mathematics of Outsourcing

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My last article on outsourcing did not provide a rigorous foundation for my negative appraisal of the outsourcing model, but there is a sound mathematical argument underlying it. So let me outline the argument. If you generate two process flow models one for internal processing and one that incorporates outsourcing of components, you will find that several additional nodes exist (usually between 4 and 6) in the outsource model. These additional nodes reflect communication, material processing, inspection, and material transfer. If we view each node as a transaction point, then node performance has a yield and variation. If success is defined as moving through each node with 100% accuracy for communication nodes and 100% yield for materials nodes, then a statistical analysis of the systems performance can be generated.

If we assume that the standard Internal Model (IM) has 100% performance, what can be expected from the more complex Outsource Model (OM).

Applying yield analysis to the Outsource Model we find:

Yield OM = (internal yield) * (external yield) = 1 * $\prod_{i=1}^{n} Xi$, where the Xi are the additional node yields in the Outsource Model

The expected yield performance of this system is:

E(YOM) = $\prod_{i=1}^{n} E(Xi)$,

The variance in yield of the system is:

V(YOM) and an approximation is given for different numbers of additional nodes (n) below:

n = 2, $v = s_2^2 s_1^2 + s_1^2 x_2^2 + s_2^2 x_1^2$

n = 3, $v = s_2^2 s_1^2 x_3^2 + s_3^2 s_1^2 x_2^2 + s_3^2 s_2^2 x_1^2 + s_1^2 (x_3 x_2)^2 + s_2^2 (x_3 x_1)^2 + s_3^2 (x_2 x_1)^2$

n = 4, $v = s_2^2 s_1^2 (x_4 x_3)^2 + s_3^2 s_1^2 (x_4 x_2)^2 + s_4^2 s_1^2 (x_3 x_2)^2 + s_1^2 (x_4 x_3 x_2)^2 +$

$s_3^2 s_2^2 (x_4 x_1)^2 + s_4^2 s_2^2 (x_3 x_1)^2 + s_2^2 (x_4 x_3 x_1)^2 + s_4^2 s_3^2 (x_2 x_1)^2 + s_3^2 (x_4 x_2 x_1)^2 + s_4^2 (x_3 x_2 x_1)^2$

where x_i is the yield of the i-th node and s_i is the standard deviation of the yield of the i-th node.

Since there are more nodes in OM than there are in IM. Assuming that production yield and variation in production yield are about the same for both systems, then the expected performance and the variation of OM and IM are given by:

E(YIM) ≥ E(YOM) and V(YIM) ≤ V(YOM)

This means that the more complex system can be expected to have lower yield and higher yield volatility.

If we assume that each time a supplier is asked to produce a part, we are making a bet_that they will be able to make and deliver a good part at the required time, then manufacturing can be viewed as a gambling game. Viewed in this light the Kelly Criterion [Kelly, 1956] can be applied. A direct result is the following corollary to the famous Gamblers Ruin theorem:

Corollary: Even in games with a positive expectation if the faction of our fortune that we bet (f) is greater than some critical value f_c, then the variance in our fortune (F) will be so large as to cause $F \rightarrow 0$ (i.e., gambler’s ruin).

So in this case a bet on either IM or OM has a positive expectation but the variance in return will generally be greater for OM. If the variance of returns is too large, then we can expect the following result. To quote Dr. Thorpe, “If a player repeatedly bets a fraction larger than f_c, then though they may temporarily experience the pleasure of a faster win rate, eventually downward fluctuations will inexorably drive their fortune toward zero” [Thorpe, 1998]. This downward trend is caused by the variance in results of the bets. The more complex OM system suffers the same problem -- higher volatility of results in our bets. If firms using OM stick to small “bets”, then they will not suffer catastrophic loss and they may actually be able to realize positive results (even results better than IM). On the other hand if they bet “too much”, the higher system variation may well cause a catastrophic loss. Unfortunately, experience indicated that most OM adopters “bet the farm”. So the only question is when will fate catch up to them?

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