



Estimating the Process Sigma by John J. Flaig, Ph.D.

Every SPC text tells practitioners that they should compute the estimated process sigma from the within subgroup sample results rather than from all the data available [Montgomery, 2001] [Duncan, 1986] [Wheeler, 1995]. In fact most of these authors warn practitioners not to estimate sigma using all the data because it may give incorrect (i.e., inflated) results. This is basically sound advice, but does following it always yield the best estimate of sigma? Let us take a closer look and see what we can deduce.

There are two commonly used ways to estimate the process sigma for individuals data -- \overline{MR}/d_2 (the recommended way) and s/c_4 (the "wrong" way) where MR is the two point moving range, s is the sample standard deviation, d_2 and c_4 are a correction factors. Let me suggest that under certain conditions the practitioner should use s/c_4 to estimate the process sigma rather than the recommended \overline{MR}/d_2 , because s/c_4 is approximately 65% more efficient in estimating sigma than is \overline{MR}/d_2 (i.e., $\text{Var}(\overline{MR}/d_2)/\text{Var}(s/c_4) \approx 1.65$, [Cryer, 1990]). Even though s is the most efficient estimator of sigma, some controversy exists as to whether to use \overline{MR}/d_2 or s/c_4 in the control limit formulas. There are reasonable arguments for both positions. Some researchers state that MR is better because it is not as biased as s, if the process mean is unstable [Wheeler, 1992] [Kamat, 1953]. On the other hand, if the process is stable, then s is better than MR particularly if the x_i 's are correlated [Cryer, 1990]. To decide which path to follow, the practitioner should consider how control limits are developed.

The process of generating control limits consists of two phases. In Phase I, the practitioner computes trial limits from a data set that is not assumed to be stable. The first step is to graph the data in time series



order of production and look for signs of instability. The next step is to test for independence, then compute \overline{MR}/d_2 and s/c_4 . The F^* test can be used to determine if a significant difference exists between these two estimators of sigma [Cruthis, 1992]. If no significant difference exists (i.e., the process is stable) and the data is not correlated, then s/c_4 should be used to compute the trial control limits. If x is stable but autocorrelated, then again using s/c_4 is recommended because \overline{MR}/d_2 will underestimate sigma. On the other hand, if there is a significant difference between \overline{MR}/d_2 and s/c_4 (i.e., the process is unstable), and the data is independent, then \overline{MR}/d_2 should be used to estimate sigma because s/c_4 will overestimate it. Finally, if the process is unstable and autocorrelated, then it should be possible to essentially eliminate autocorrelation through strategic sampling. Hence \overline{MR}/d_2 should be used on the reduced data set to estimate sigma. After a decision is made on which is the best sigma estimator, then trial control limits can be computed and the chart can be examined for signs of instability. Once the points causing instability are identified, then the practitioner can proceed to Phase II.

In Phase II, the unstable points with assignable causes are removed from the data set. Hence, the remaining data is assumed to be from a process that is stable, and therefore the recomputed control limits should be based on s/c_4 rather than \overline{MR}/d_2 .

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