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The Relation Between Cpk and Fraction Nonconforming

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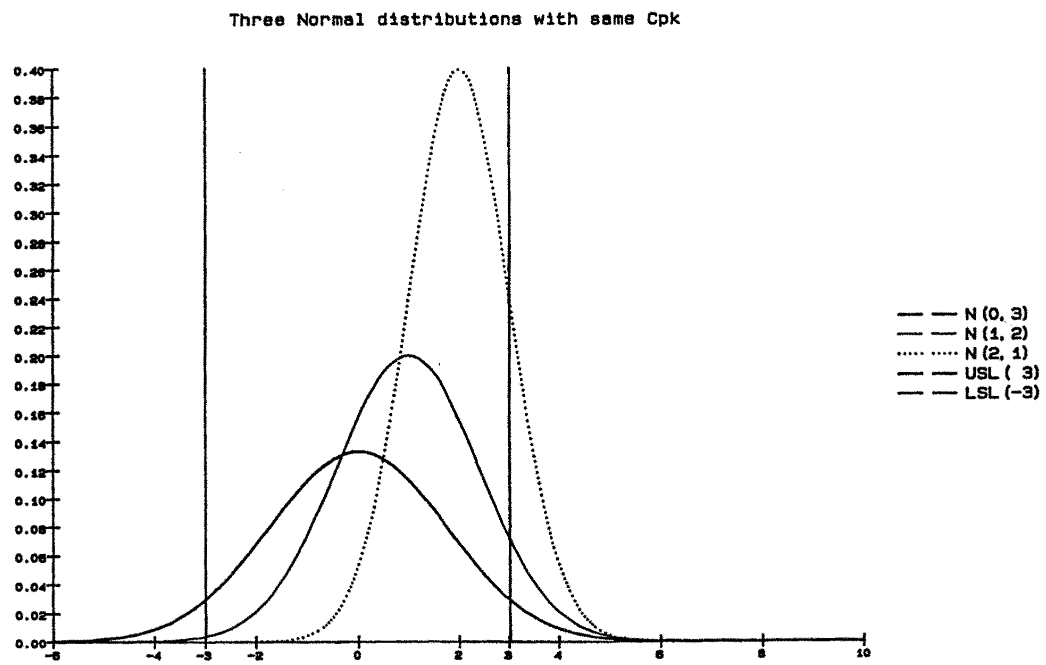
Dr. Montgomery's textbook Introduction to Statistical Process Control has several problems dealing with capability analysis [Montgomery, 2001]. For example, the following answer is given for problem 7-8. Process A has a mean of 2000 and standard deviation of 14.27 whereas Process B has a mean of 2100 with a standard deviation of 4.76. Process B will result in fewer defective assemblies. For the parts $Cpk_A = 1.045 < 1.566 = Cpk_B$ indicates that more parts from Process B are within specification than from Process A.

This argument implies that if the capability indices are different, then the process with the larger Cpk index will produce few parts that are out of specification. This statement translates into the logical expression:

$$Cpk_A \leq Cpk_B \Rightarrow p_B \leq p_A$$

where the p's represent the fraction nonconforming.

In many cases this relationship may be true but is it true in general? To answer this question, consider the following figure:



Let N(1, 2) be the “A” distribution and N(2, 1) be the “B”. In this case $Cpk_A = Cpk_B = 1/3$, but it is easy to see that $p_B > p_A$. This counter example shows that the logical argument given in the answer book to Montgomery’s text is incorrect. Hence, just because Cpk_A is less than Cpk_B this does NOT imply that the fraction nonconforming generated by Process "B" will be less than that generated by Process “A”.

Montgomery, D. C. (2001). Introduction to Statistical Quality Control. 4 Ed., John Wiley and Sons, New York, NY.

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